Machine Learning for Symbolic Integration Algorithm Selection

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Motivation - Maple and Machine Learning

- Computer Algebra refers to the study and development of algorithms and software for manipulating mathematical expressions and other mathematical objects

- As a Computer Algebra System, Maple should always return the correct answer
  - Alternatively, Maple shouldn't output anything at all if there is no answer or it cannot compute one!

- Machine Learning has seen many applications in various fields. Computer Algebra is now starting to catch up.

- A problem exists between Computer Algebra and Machine Learning
  - E.g. I build a model that has 99% accuracy for computing an integral given an expression. Is this acceptable?

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Machine Learning and Integration

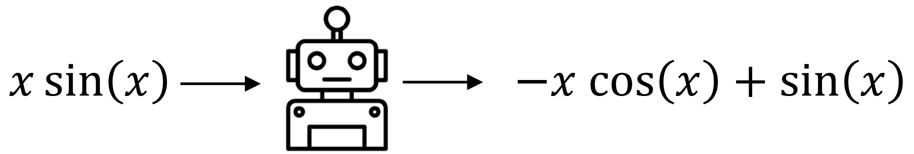
- Two approaches:

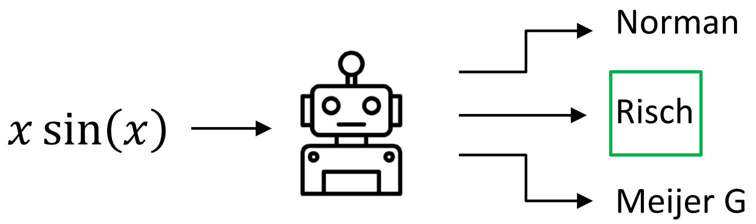
Directly solving a problem

- Compute the result of a task given an input
- E.g. Given an expression, calculate its integral
- Performance based on accuracy

Algorithm Selection

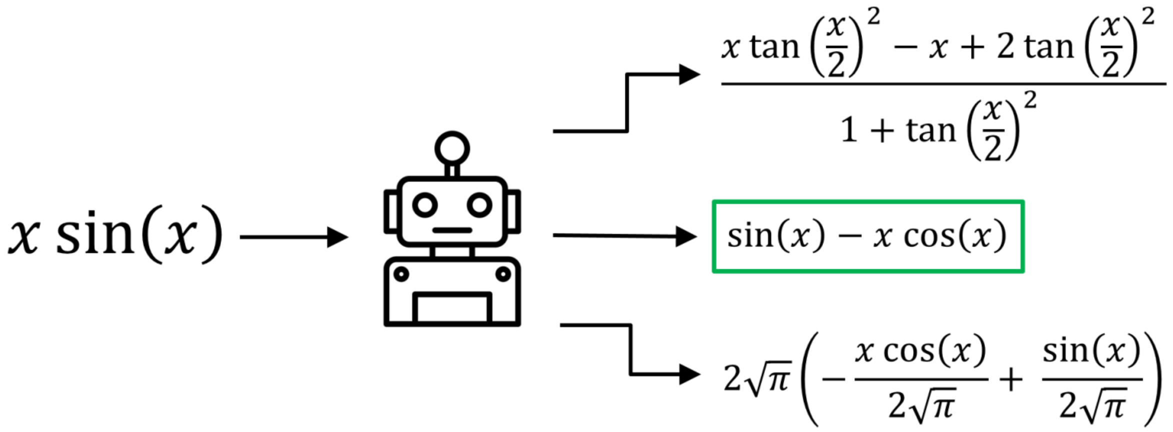
- If an algorithm can make an arbitrary choice, use ML to help guide that choice
- E.g. Given an expression, which integration rule should we first try
- Performance based on speed & output quality





Objective

- There are two objective functions we can consider when assessing how well a sub-algorithm does
  - Output length
  - Runtime



- Sub-algorithms selected are the ones that outputs the shortest expression.
  - Could be that a sub-algorithm was successful but gave a longer answer so we consider that a fail

- Sub-algorithms are not mutually exclusive

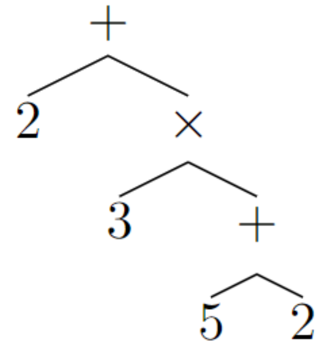
Generating Data – Random Expressions

Deep Learning for Symbolic Mathematics - Lample G, Charton F (Meta AI research)

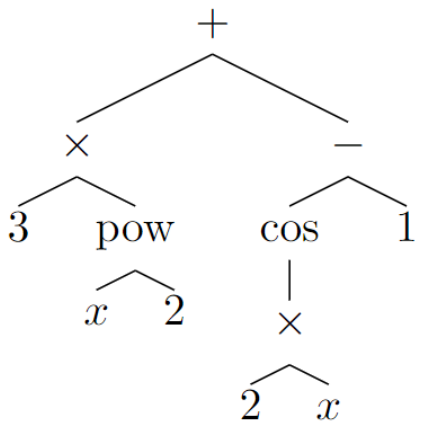
Mathematical expressions can be represented as trees:

- operators and functions as internal nodes
- numbers, constants and variables as leaves

$$2 + 3 \times (5 + 2)$$



$$3x^2 + \cos(2x) - 1$$



Generating Data – (Integrand, Integral) pairs

Deep Learning for Symbolic Mathematics - Lample G, Charton F (Meta AI research)

– FWD: Integrate an expression f through a CAS to get F and add the pair (f, F) to the dataset.

– BWD: Differentiate an expression f to get f' and add the pair (f', f) to the dataset.

– IBP: Given two expressions f and g, calculate f' and g'. If $\int f'g$ is known then the following holds (integration-by-parts):

$$\int fg' = fg - \int f'g.$$

Thus we add the pair $(fg', fg - \int f'g)$ to the dataset.

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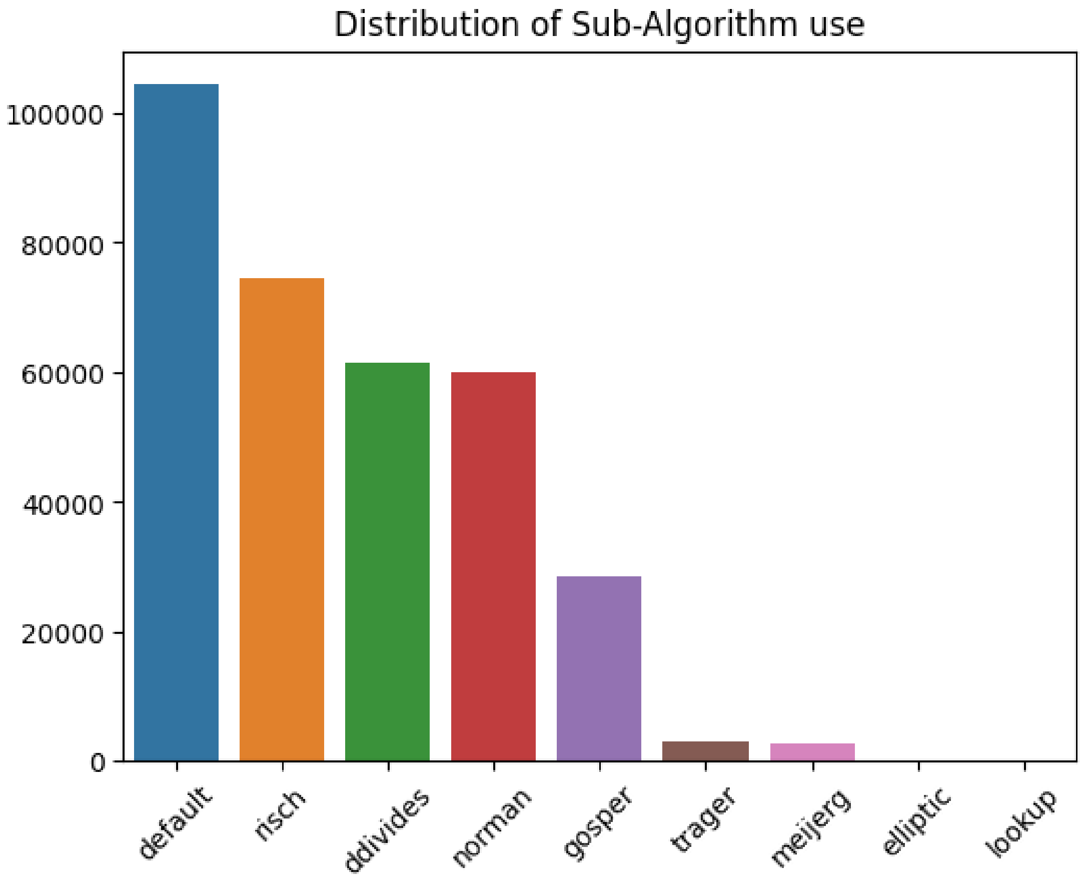
The Dataset

- FWD
- BWD
- IBP

(FWD, BWD, IBP) Lample & Charton (2020)

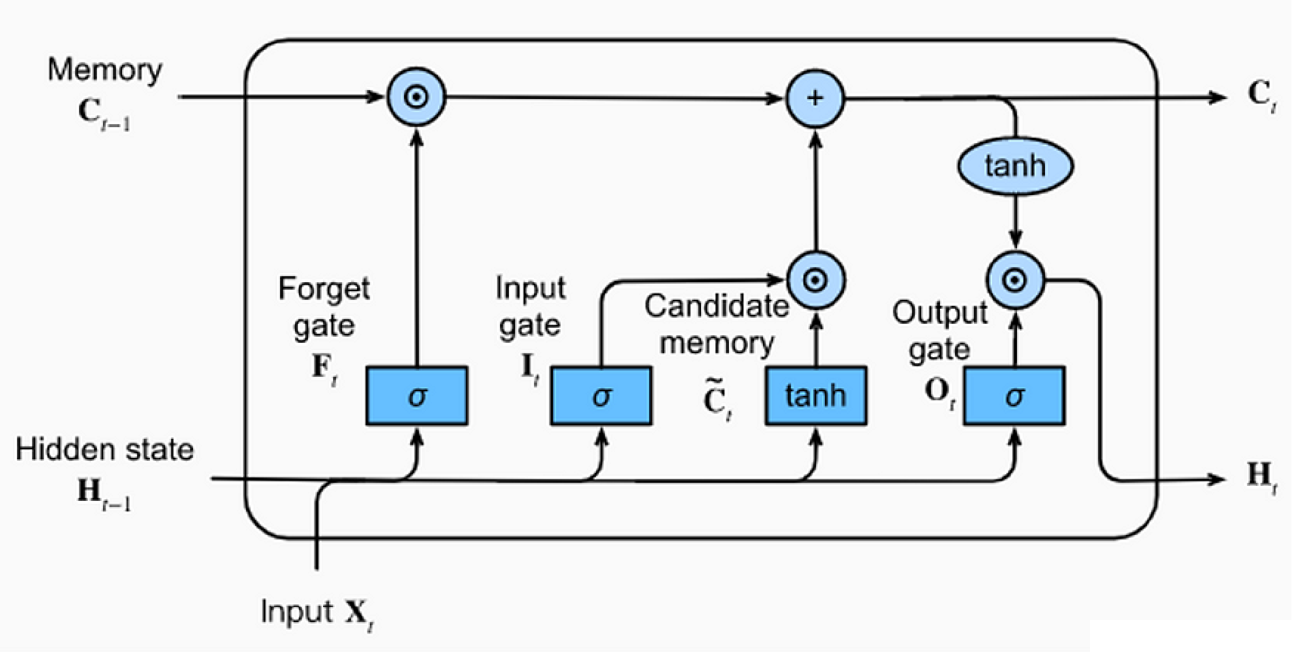
- Risch Method – Barket et al. (2023)
- The Substitution Rule

$$\int f(g(x))g'(x)\,dx$$



LSTMs

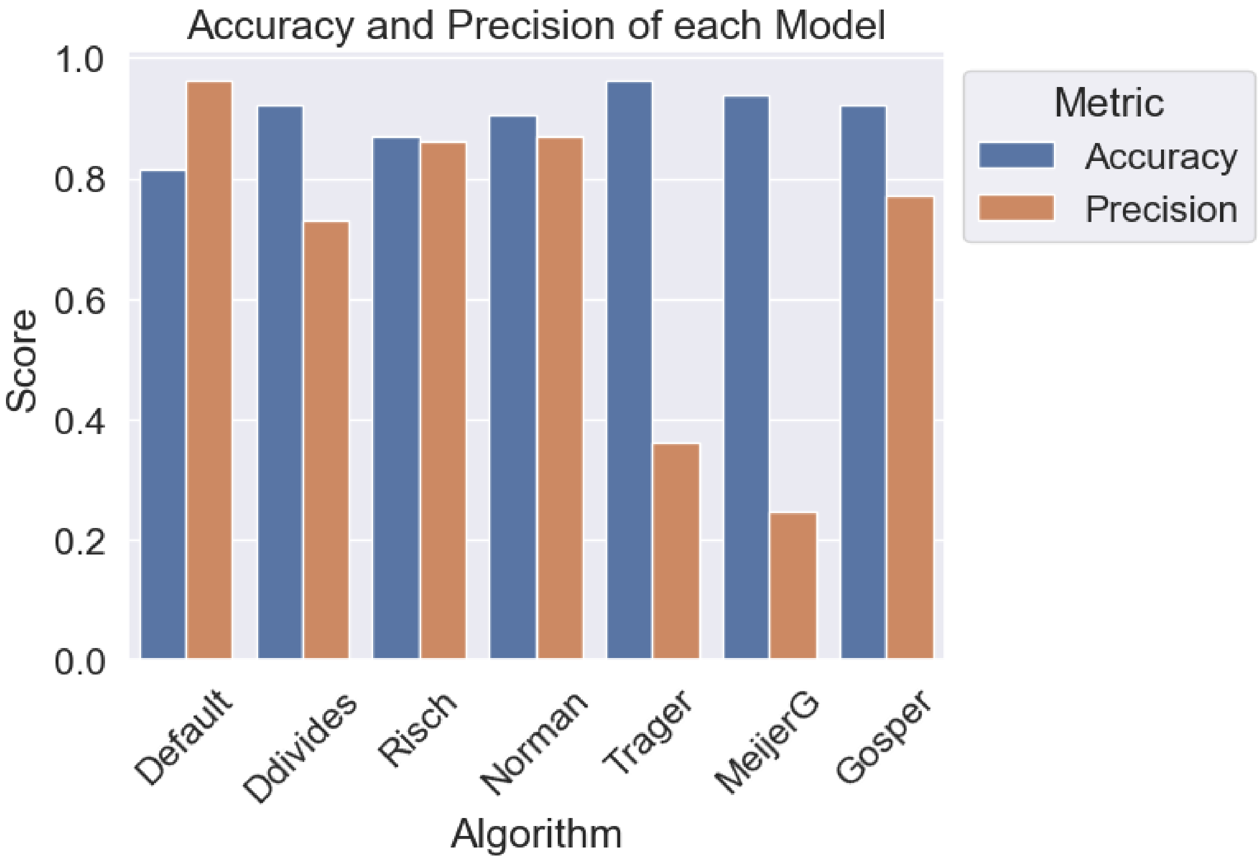
- LSTM = Long Short-Term Memory
- A Neural Network architecture for handling sequence data (text, time series, etc.)
- Able to remember information far in the past (Long term memory) as well as use the information near the current step (short term memory)
- Performs much better than vanilla neural networks for tasks such as text classification, language translation, and time series predictions

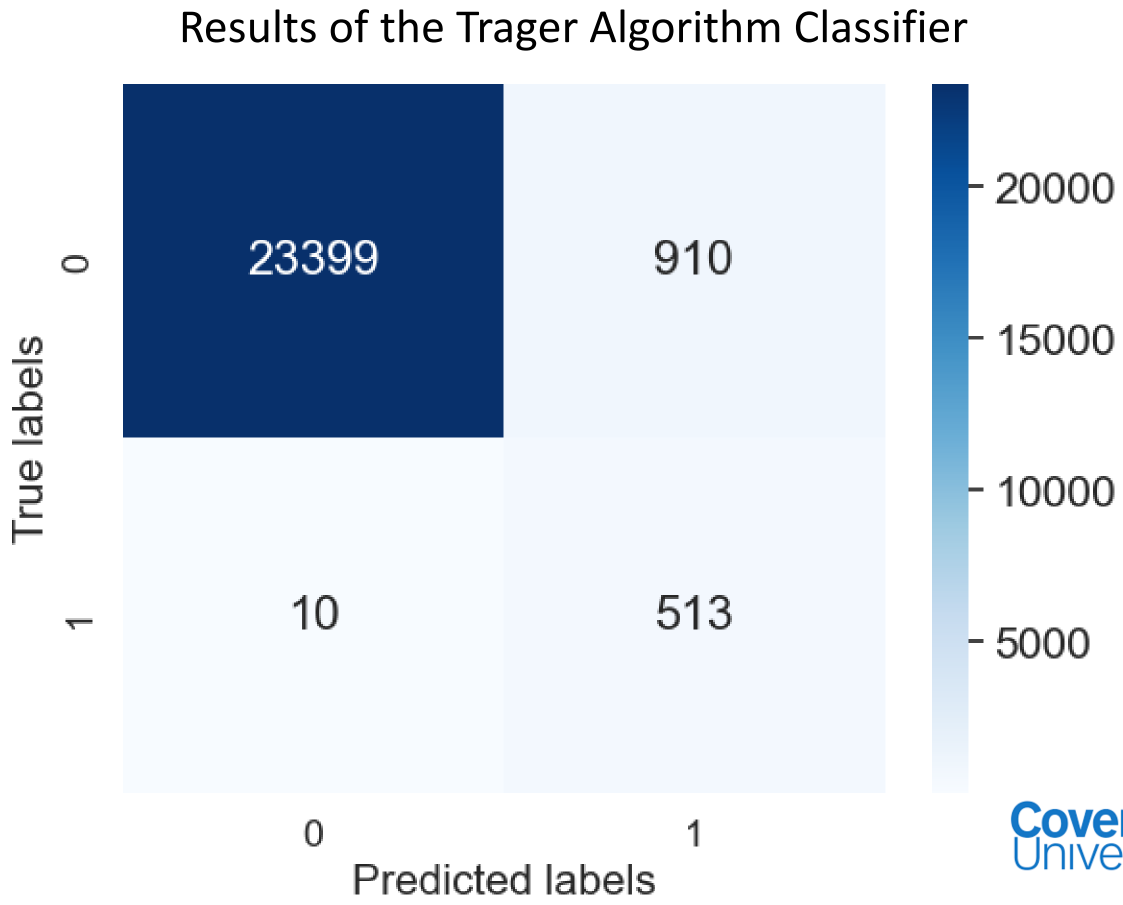


$$\text{Accuracy} = \frac{\text{True Positives} + \text{True Negatives}}{\text{Total}}$$

Initial Results

$$\text{Precision} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Positives}}$$





Comparison against Maple

- We trained our ML model on FWD, IBP, Risch, and Sub data to predict the sub-algorithm with the smallest output
- The model is tested with 25,000 integrable expressions

	Maple	LSTM	Tie
Same Data Generation Methods	2,147	8,746	14,107
BWD	2,437	1,482	21,081

- Suggests bias in the dataset

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```
> f := 1 - x*cos(cos(x))*sin(x) + sin(cos(x))
```

$$f := 1 - x\cos(\cos(x))\sin(x) + \sin(\cos(x))$$

```
> # Maple's answer
  int(f, x)
```

$$\frac{x + x\tan\left(\frac{x}{2}\right)^2 + x\tan\left(\frac{1-\tan\left(\frac{x}{2}\right)^2}{2\left(1+\tan\left(\frac{x}{2}\right)^2\right)}\right)^2 + x\tan\left(\frac{x}{2}\right)^2\tan\left(\frac{1-\tan\left(\frac{x}{2}\right)^2}{2\left(1+\tan\left(\frac{x}{2}\right)^2\right)}\right)^2 + 2x\tan\left(\frac{1-\tan\left(\frac{x}{2}\right)^2}{2\left(1+\tan\left(\frac{x}{2}\right)^2\right)}\right) + 2x\tan\left(\frac{x}{2}\right)^2\tan\left(\frac{1-\tan\left(\frac{x}{2}\right)^2}{2\left(1+\tan\left(\frac{x}{2}\right)^2\right)}\right)}{\left(1+\tan\left(\frac{1-\tan\left(\frac{x}{2}\right)^2}{2\left(1+\tan\left(\frac{x}{2}\right)^2\right)}\right)^2\right)\left(1+\tan\left(\frac{x}{2}\right)^2\right)}$$

```
> # ML-suggested answer
  int(f, x, method=risch)
```

$$x + \sin(\cos(x))\,x$$

Thank you! Questions?