

# Decomposing Truth

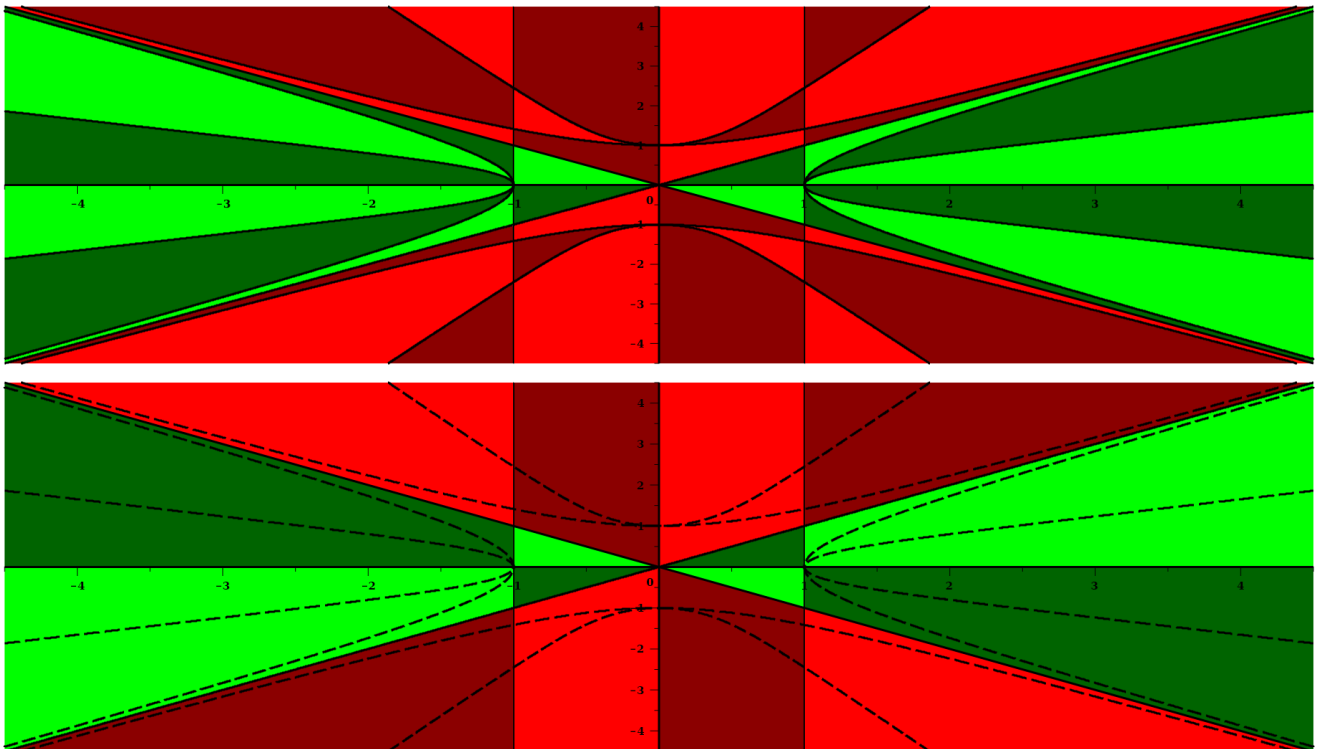
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Maple Conference 2021

The images below concern this potential simplification that a Maple user could make:

$$\begin{aligned} &> \text{sqrt}(z^2-1)*\text{sqrt}(z^2+1) = \text{sqrt}(z^4-1); \\ &\quad \sqrt{z^2-1} \sqrt{z^2+1} = \sqrt{z^4-1} \end{aligned}$$

While valid for real  $z$  this identity does not hold for some complex values of  $z$ . The green areas of the images show where it is true and the red areas where it is false (the image does not clarify the truth on the boundaries). The images were produced by decomposing the complex plane according to the branch cuts of the functions in the identity, which may be obtained from Maples FunctionAdvisor tool [1]. Both images are cylindrical algebraic decompositions of these polynomials, with the second showing a more efficient decomposition from the algorithm in [2].



- [1] M. England, E. Cheb-Terrab, R. Bradford, J.H. Davenport and D. Wilson. Branch cuts in Maple 17. ACM Communications in Computer Algebra 48:1 (issue 187), pp. 24-27, ACM, 2014.  
<http://dx.doi.org/10.1145/2644288.2644293>
- [2] R. Bradford, J.H. Davenport, M. England, S. McCallum and D. Wilson. Truth table invariant cylindrical algebraic decomposition. J. of Symbolic Computation 76, pp. 1-35, 2016.  
<http://dx.doi.org/10.1016/j.jsc.2015.11.002>