Heuristics in SMT Solving: To Learn or not to Learn?

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RWTH Aachen University, Germany

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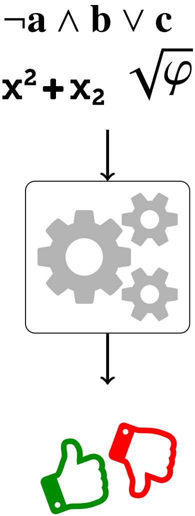
What is this talk about?



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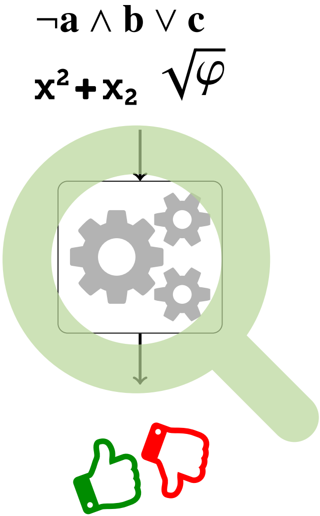

What is the role of heuristics in SMT solving?

Are there potentials for learning?

The satisfiability problem

Propositional logic

Formula: $(a \vee \neg b) \wedge (\neg a \vee b \vee c)$

Satisfying assignment: $a = \textit{true}$, $b = \textit{false}$, $c = \textit{true}$

It is perhaps the most well-known NP-complete problem [Cook'71].

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Non-linear real algebra (NRA)

Formula: $(x - 2y > 0 \vee x^2 - 2 = 0) \wedge x^4 y + 2x^2 - 4 > 0$

Satisfying assignment: $x = \sqrt{2}$, $y = 2$

There are some hard problem classes... non-linear integer arithmetic is even undecidable.

SAT solving for propositional logic

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- dynamic variable ordering (VSIDS)
- resolution driven by enumeration/propagation search, learning resolvents (CDCL), forgetting learnt clauses

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Problem: how to extract characteristic information for training sets?

(Full/less) lazy SMT solving

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φ quantifier-free FO formula

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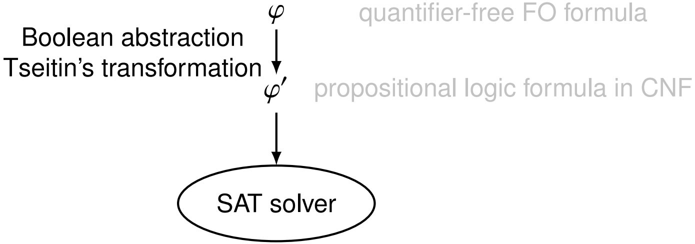
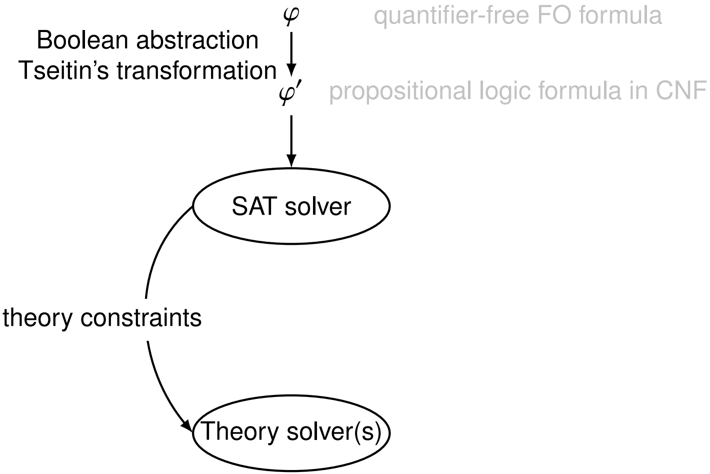
Boolean abstraction
Tseitin's transformation
$\downarrow$

φ' propositional logic formula in CNF

(Full/less) lazy SMT solving



(Full/less) lazy SMT solving



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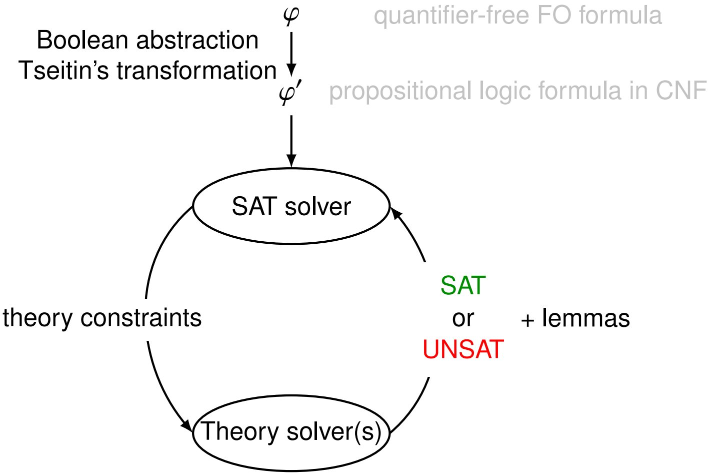

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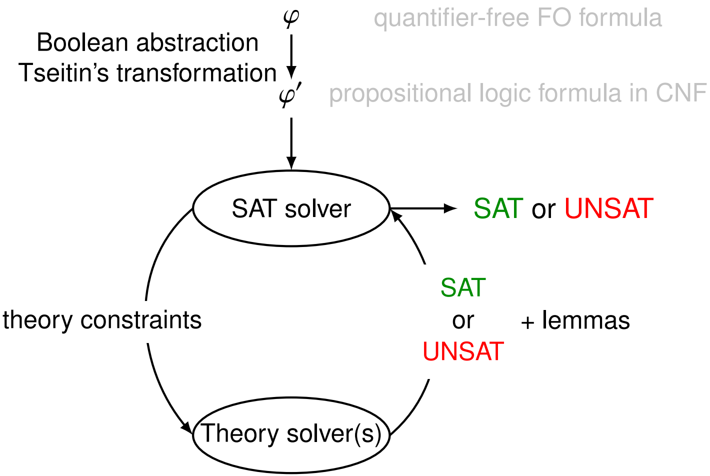

General SMT solving heuristics

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- A particularly interesting case: variable ordering.

  In SAT solving, VSIDS is very successful, but the variable odering at the Boolean level is not connected to the theory solver.

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- Other cases: preprocessing, restarts, constraint ordering, value ordering, ...

Some SMT solvers for arithmetic theories

- AProVE (RWTH Aachen University, Germany)
- CVC4 (New York and Iowa, USA)
- MathSAT 5 (FBK, Italy)
- MiniSmt (University of Innsbruck, Austria)
- Boolector (JKU, Austria)
- SMT-RAT (RWTH Aachen University, Germany)
- veriT+Redlog (CNRS Inria, France and MPI Informatics, Germany)
- Z3 (NYU, Microsoft Research, USA)
- Yices 2 (SRI International, USA)
- ...

Our SMT-RAT library SAT'12, SAT'15

SMT solver

Strategic composition of SMT-RAT modules

SMT real-algebraic toolbox

collection of solver modules

CArL

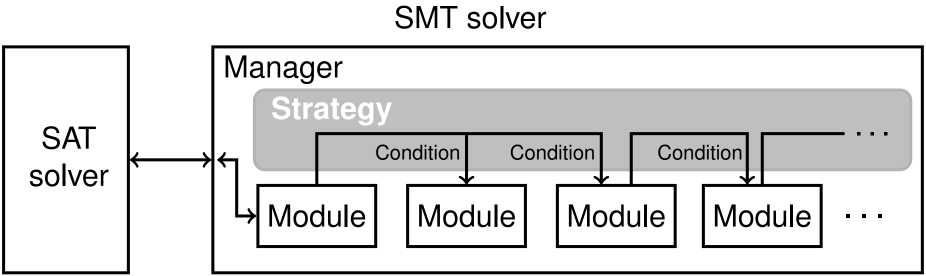
real-arithmetic computations

gmp, Eigen3, boost

- MIT licensed source code: `github.com/smtrat/smtrat`
- Documentation: `smtrat.github.io`

Strategic composition of solver modules in SMT-RAT

- Strategy: directed graph over modules with guarded edges
- Guard: may talk about the formula forwarded to backends
- Backend-calls: passed to all enabled successors → parallelism



Solver modules in SMT-RAT

CArL library for basic arithmetic datatypes and computations [NFM'11, CAI'11, Sapientia'18]

Basic modules

- SAT solver
- CNF converter
- Preprocessing/simplifying modules

Non-algebraic decision procedures

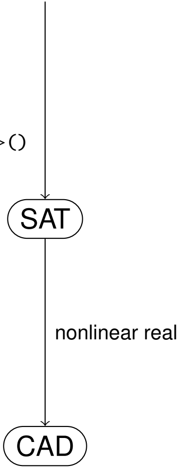
- Bit-vectors
- Bit-blasting
- Equalities and uninterpreted functions
- Pseudo-Boolean formulas
- Interval constraint propagation

Algebraic decision procedures

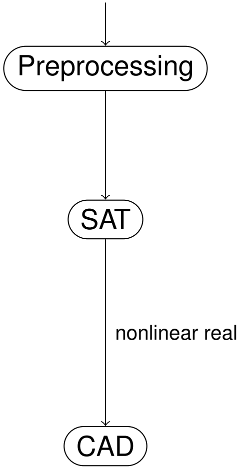
- Fourier-Motzkin variable elimination
- Simplex
- Subtropical satisfiability
- Gröbner bases [CAI'13]
- MCSAT (FM,VS,CAD)
- Cylindrical algebraic decomposition [CADE-24, SC^2'17, PhD Loup, PhD Kremer]
- Virtual substitution [FCT'11, SC^2'17, PhD Corzilius]
- Generalized branch-and-bound [CASC'16]
- Cube tests

SMT-RAT strategies

```
class myStrategy: public Manager {
  myStrategy(): Manager() {
    setStrategy(
      addBackend<SATModule<SATSettings>>(
        addBackend<CADModule<CADSettings>>()
      )
    );
  }
};
```



SMT-RAT strategies



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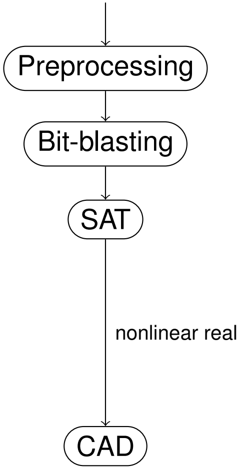

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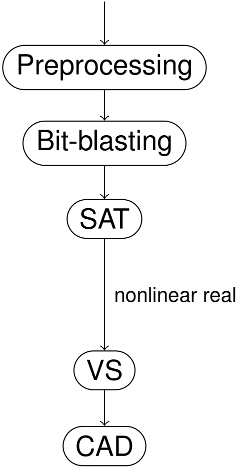

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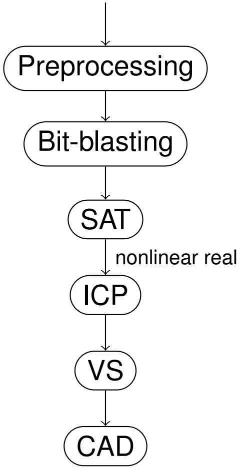

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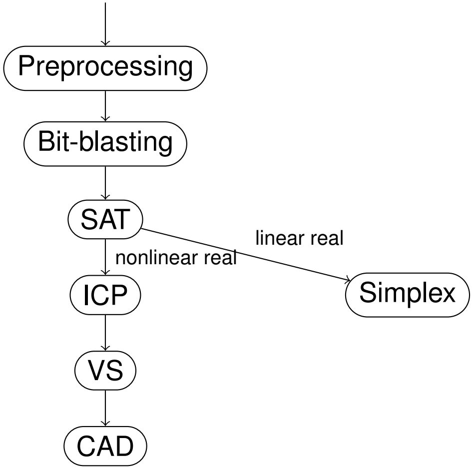

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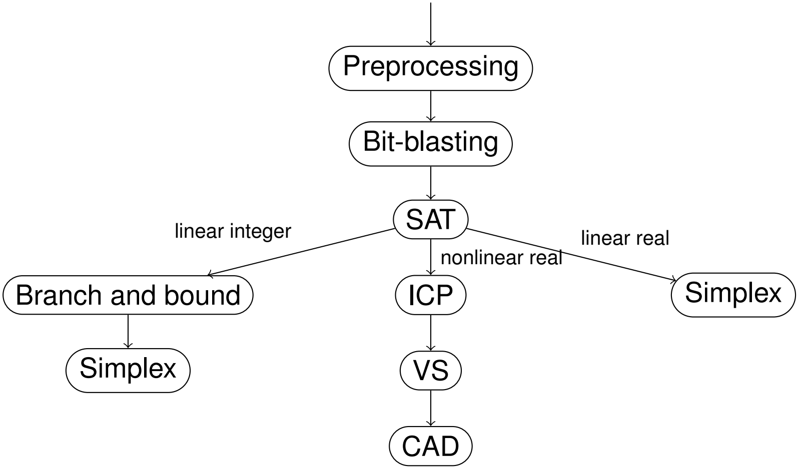

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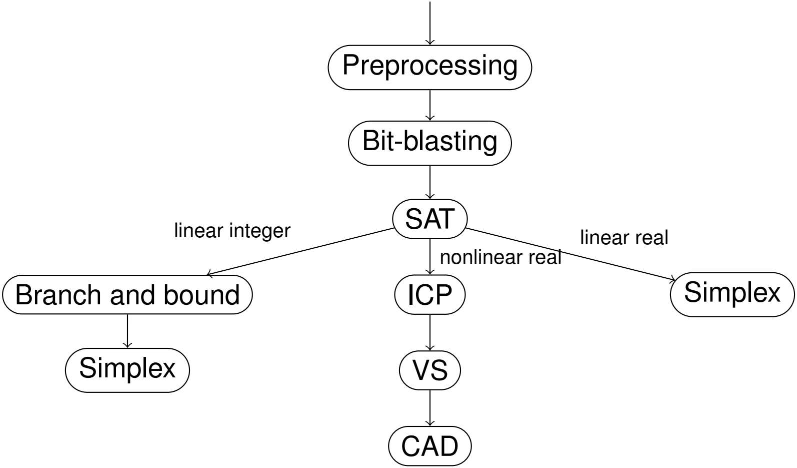
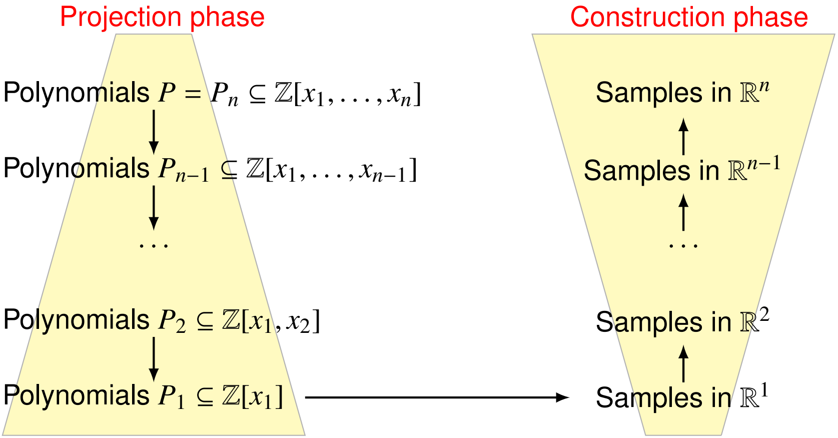
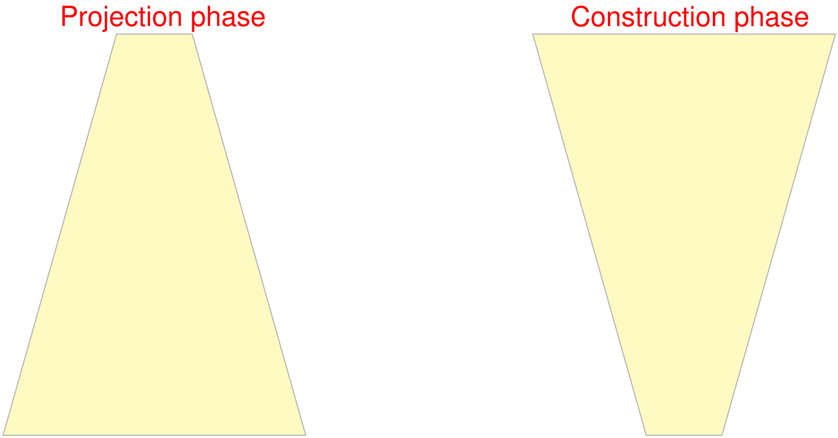


Which strategy to use for which problem?

The cylindrical algebraic decomposition (CAD) method



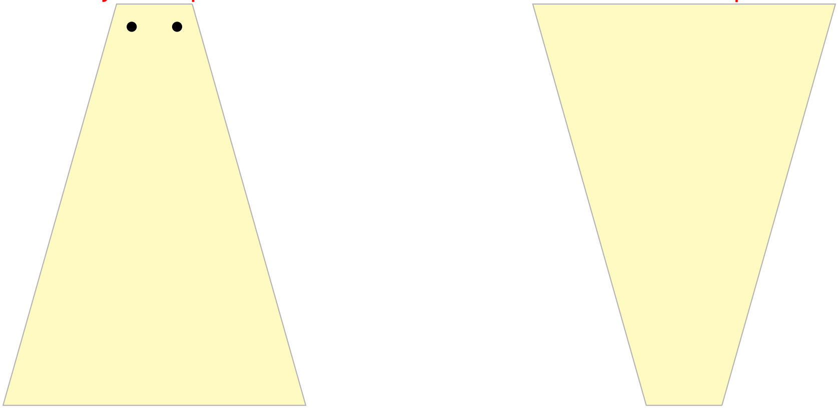
Heuristics in the CAD method



Heuristics in the CAD method

Projection phase

Construction phase



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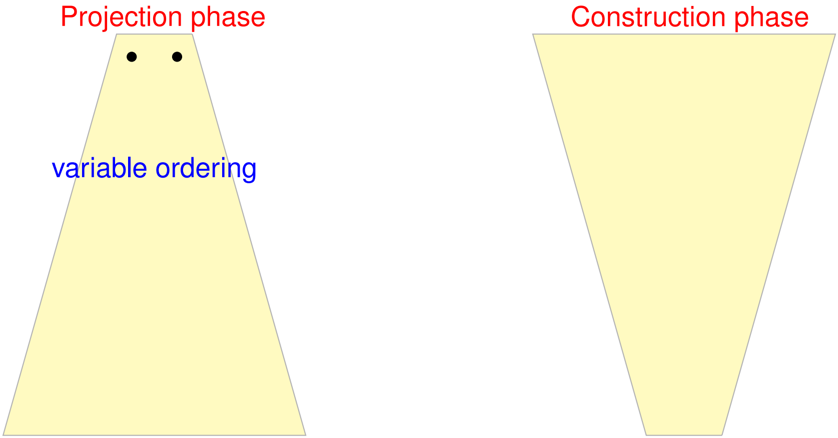

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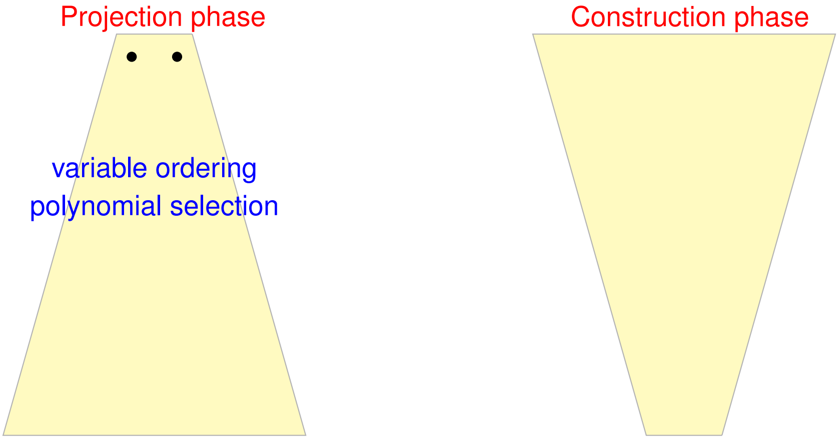

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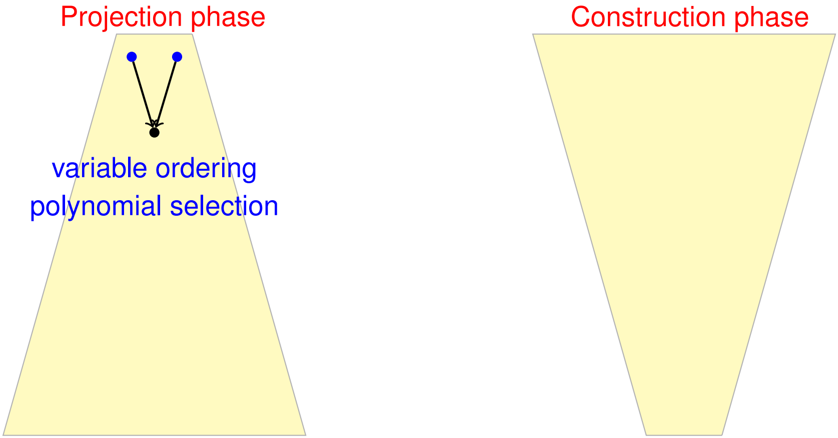

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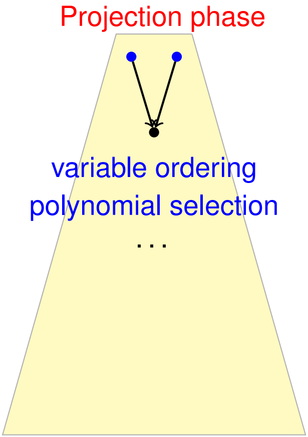

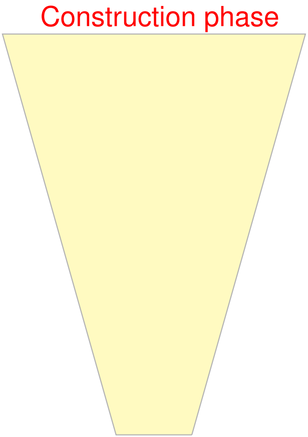

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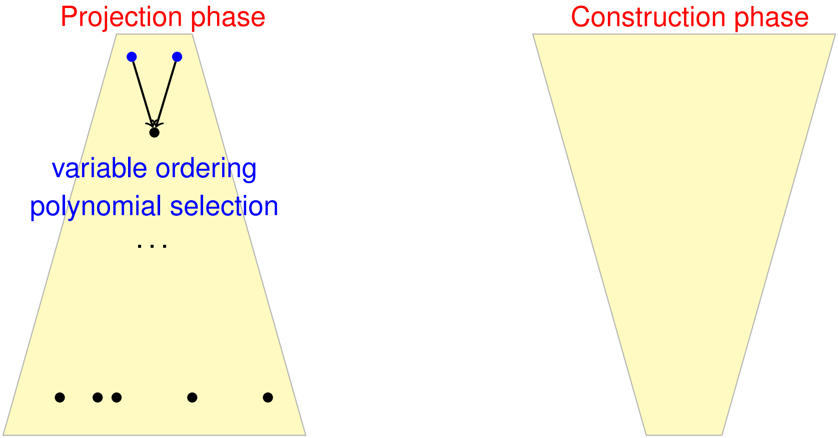

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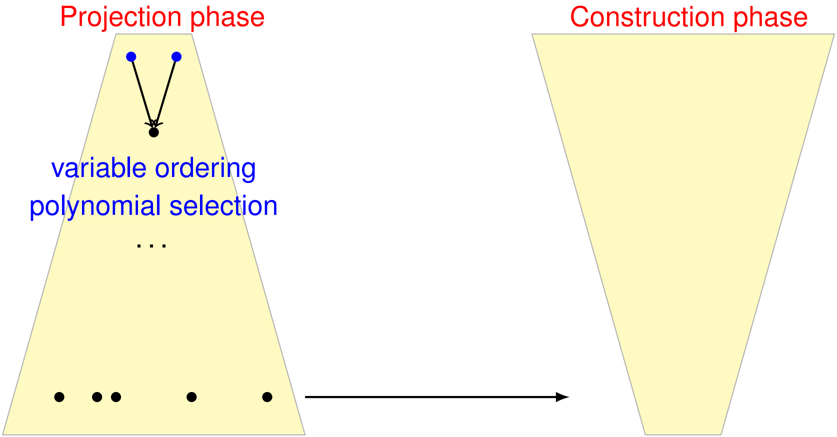

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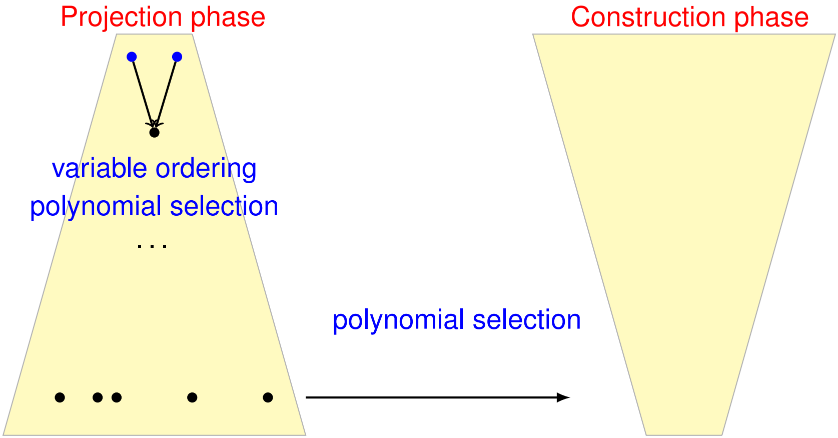

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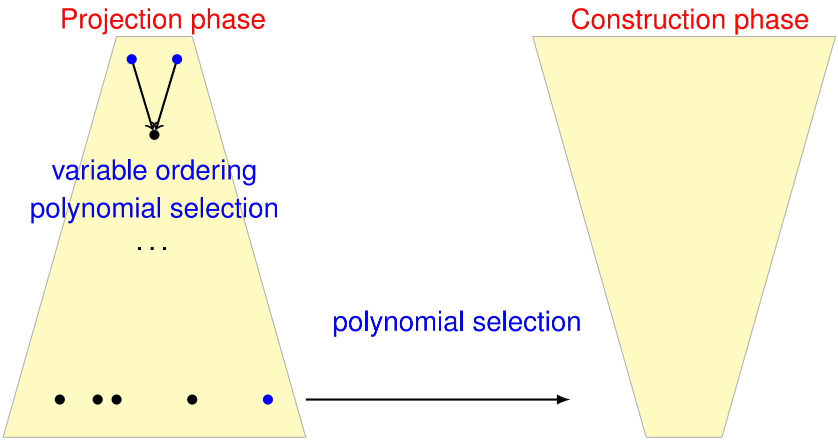

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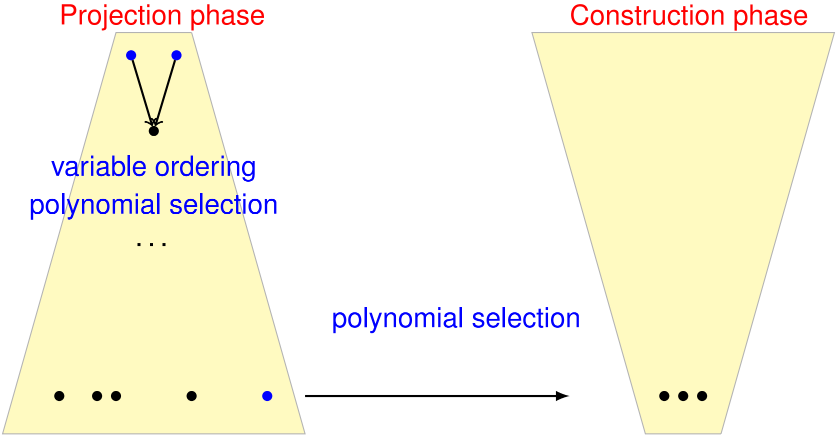

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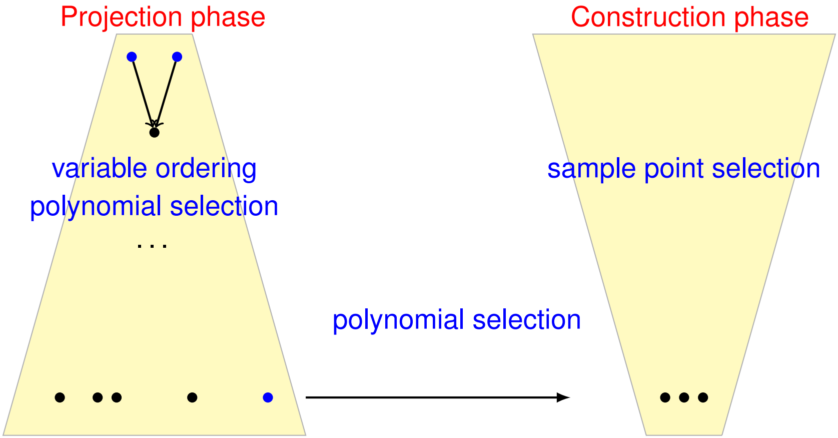

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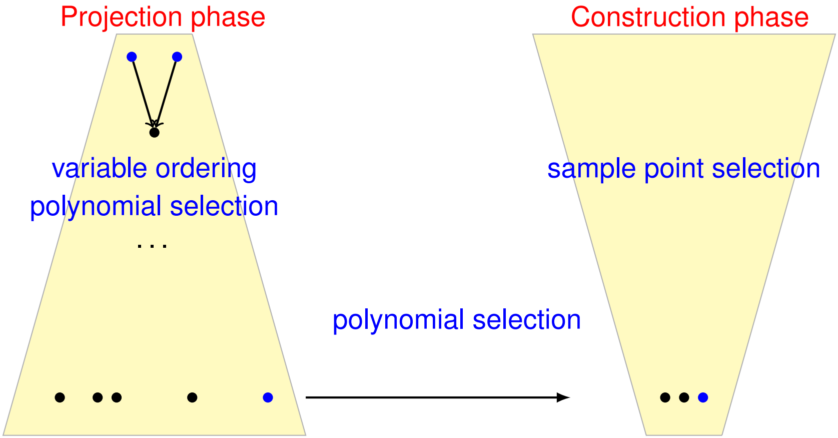

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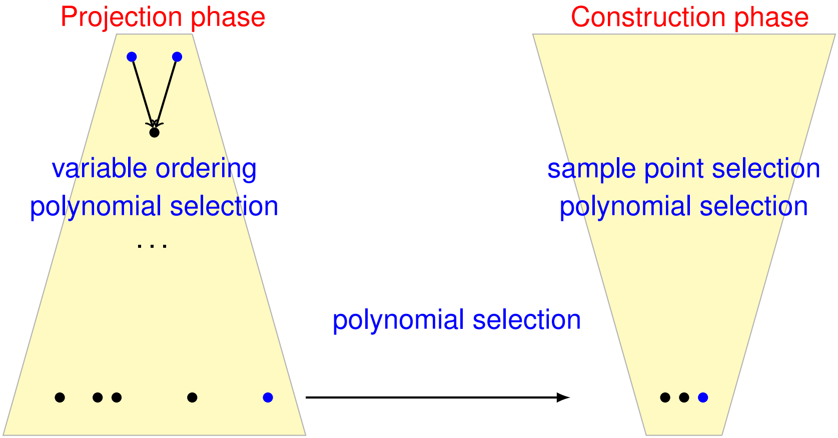

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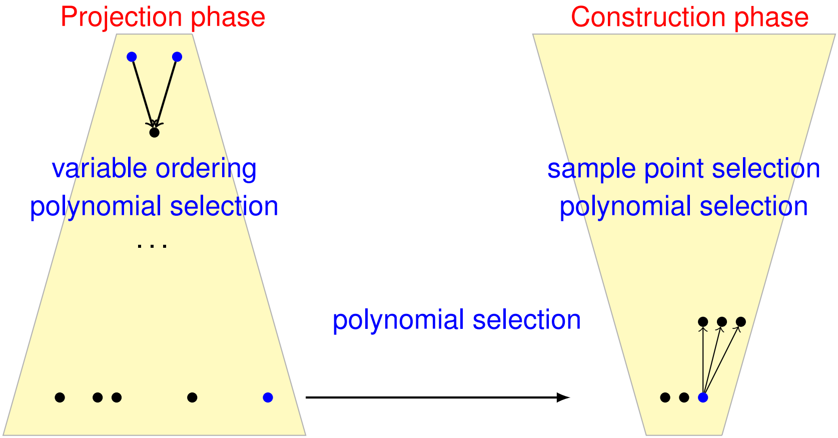

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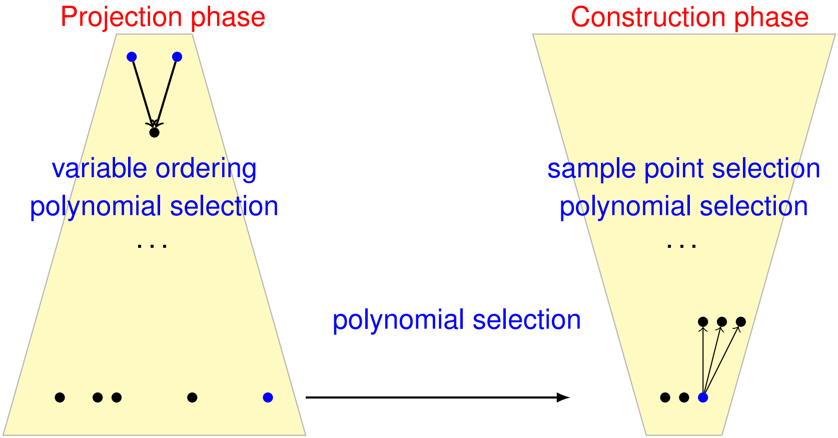

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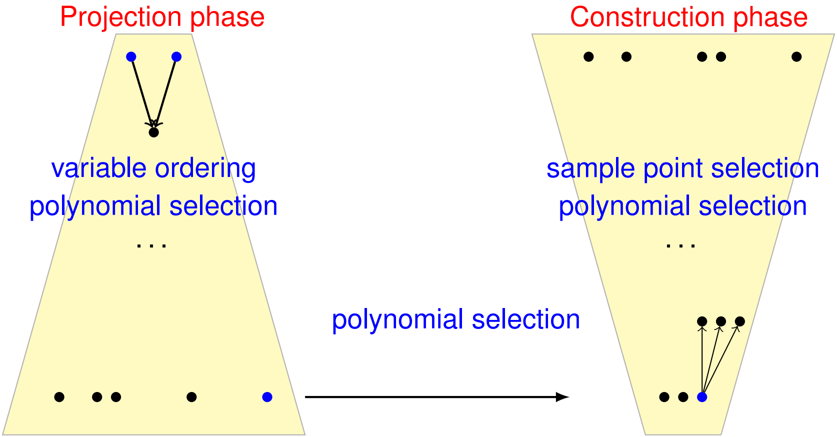

Experimental results: Projection order

11354 `QF_NRA` benchmarks, TO: 60 secs

- C: complexity
- S: single projection
- P: paired projection
- L_i: level in increasing order
- L_d: level in decreasing order

Solver	solved		runtime
CAD C	8075	71.1 %	1.13
CAD SC	8074	71.1 %	1.13
CAD PC	8076	71.1 %	1.12
CAD L_iC	**8135**	71.6 %	1.28
CAD L_dC	**8135**	71.6 %	1.18

Experimental results: Sampling

11354 QF_NRA benchmarks, TO: 60 secs

Solver	solved		average runtime
CAD midpoint	8147	71.8 %	1.21
CAD int closest to midpoint	8155	71.8 %	1.19
CAD smallest int	**8158**	71.9 %	1.22
CAD largest int	8144	71.7 %	1.20
CAD int close to 0	8154	71.8 %	1.20
CAD int far from 0	8146	71.7 %	1.21

Experimental results: Lifting

11354 QF_NRA benchmarks, TO: 60 secs

T: type (integer, rational, algebraic)

S: size

A: absolute value

L: level

Solver	solved		average runtime
CAD TSA	8118	71.5 %	1.21
CAD S	8121	71.5 %	1.22
CAD T	8138	71.7 %	1.20
CAD LTS	8143	71.7 %	1.22
CAD LT	**8144**	71.7 %	1.20

An attempt for a conclusion

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There are probably great potentials in learning heuristics, but a number of problems need to be solved before we can explore these possibilities.