Efficient Subformula Orders for Real Quantifier Elimination of Non-prenex Formulas

Munehiro Kobayashi[1,2] Hidenao Iwane[3]
Takuya Matsuzaki[4] Hirokazu Anai[2,5]

[1]Schilf Institute Co., Ltd., Japan

[2]University of Tsukuba, Japan

[3]Fujitsu Laboratories Ltd., Japan

[4]Nagoya University, Japan

[5]Kyushu University, Japan

This talk is based on the paper[1].

[1] Kobayashi, M., Iwane, H., Matsuzaki, T., Anai, H.: Efficient subformula orders for real quantifier elimination of non-prenex formulas. In: International Conference on Mathematical Aspects of Computer and Information Sciences. pp. 236–251. Springer (2015)

Outline

Acknowledgements

Todai robot project[2,3] arranged the research environment and made a financial support.

Schilf Institute Co., Ltd. provided the travel expenses.

---

[2] Arai, N.H., Matsuzaki, T., Iwane, H., Anai, H.: Mathematics by machine. In: Proceedings of the 39th International Symposium on Symbolic and Algebraic Computation. pp. 1–8. ACM (2014)

[3] Matsuzaki, T., Iwane, H., Anai, H., Arai, N.H.: The most uncreative examinee: a first step toward wide coverage natural language math problem solving. In: Twenty-Eighth AAAI Conference on Artificial Intelligence. pp. 1098–1104 (2014)

Outline

Goal

- We aim at increasing the number of QE problems for *non-prenex formulas* solved in 600 seconds by modifying the order of processing subformulas.
- We investigate the possibility of automatizing manual design for effective heuristics that requires trials and errors.

Prenex Formula

$$\exists x_0 \exists x_1 \exists x_2 \exists x_3 \exists x_4 \exists x_5 \Big($$

$$(-x_0 \leq -1) \vee$$
$$(x_1 \leq 0) \vee$$
$$\Big(((x_2^3 x_4 x_5 - x_2^2 x_4 - x_2^2 x_5 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_4 x_5 x_6 - x_2^2 x_4 x_5 - x_2^2 x_4 x_6 - x_2^2 x_5 x_6 + x_2 x_4 + x_2 x_5 + x_2 x_6 - x_6 = 0) \ \wedge$$
$$(x_2^3 x_4 x_6 - x_2^2 x_4 - x_2^2 x_6 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_4 x_5 x_6 - x_2^2 x_4 x_5 - x_2^2 x_4 x_6 - x_2^2 x_5 x_6 + x_2 x_4 + x_2 x_5 + x_2 x_6 - x_5 = 0)) \ \wedge$$
$$(x_2^3 x_3 - x_2^2 x_3 - x_2^2 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_3 x_4 - x_2^2 x_3 x_4 - x_2^2 x_3 - x_2^2 x_4 + x_2 x_3 + x_2 x_4 + x_2 - x_4 = 0) \ \wedge$$
$$(x_2^3 x_4 - x_2^2 x_4 - x_2^2 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_3 x_4 - x_2^2 x_3 x_4 - x_2^2 x_3 - x_2^2 x_4 + x_2 x_3 + x_2 x_4 + x_2 - x_3 = 0)\Big)$$
$$\Big).$$

Prenex Formula

$$\exists x_0 \exists x_1 \exists x_2 \exists x_3 \exists x_4 \exists x_5 \Big($$

$$(-x_0 \le -1) \vee$$
$$(x_1 \le 0) \vee$$
$$\Big(((x_2^3 x_4 x_5 - x_2^2 x_4 - x_2^2 x_5 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_4 x_5 x_6 - x_2^2 x_4 x_5 - x_2^2 x_4 x_6 - x_2^2 x_5 x_6 + x_2 x_4 + x_2 x_5 + x_2 x_6 - x_6 = 0) \wedge$$
$$(x_2^3 x_4 x_6 - x_2^2 x_4 - x_2^2 x_6 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_4 x_5 x_6 - x_2^2 x_4 x_5 - x_2^2 x_4 x_6 - x_2^2 x_5 x_6 + x_2 x_4 + x_2 x_5 + x_2 x_6 - x_5 = 0)) \wedge$$
$$(x_2^3 x_3 - x_2^2 x_3 - x_2^2 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_3 x_4 - x_2^2 x_3 x_4 - x_2^2 x_3 - x_2^2 x_4 + x_2 x_3 + x_2 x_4 + x_2 - x_4 = 0) \wedge$$
$$(x_2^3 x_4 - x_2^2 x_4 - x_2^2 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_3 x_4 - x_2^2 x_3 x_4 - x_2^2 x_3 - x_2^2 x_4 + x_2 x_3 + x_2 x_4 + x_2 - x_3 = 0)\Big)$$

$$\Big).$$

Non-prenex Formula

$$\varphi \equiv \varphi_1 \vee \varphi_2 \vee \varphi_3,$$

where

$$\varphi_1 \equiv \exists x_0 (-x_0 \leq -1),$$
$$\varphi_2 \equiv \exists x_1 (x_1 \leq 0),$$
$$\begin{aligned}\varphi_3 \equiv \exists x_2 \exists x_3 \exists x_4 \Big(\exists x_5 \big((x_2^3 x_4 x_5 - x_2^2 x_4 - x_2^2 x_5 + x_2 + 1 = 0 \vee \\ x_2^3 x_4 x_5 x_6 - x_2^2 x_4 x_5 - x_2^2 x_4 x_6 - x_2^2 x_5 x_6 + x_2 x_4 + x_2 x_5 + x_2 x_6 - x_6 = 0) \wedge \\ (x_2^3 x_4 x_6 - x_2^2 x_4 - x_2^2 x_6 + x_2 + 1 = 0 \vee \\ x_2^3 x_4 x_5 x_6 - x_2^2 x_4 x_5 - x_2^2 x_4 x_6 - x_2^2 x_5 x_6 + x_2 x_4 + x_2 x_5 + x_2 x_6 - x_5 = 0)\big) \wedge \\ (x_2^3 x_3 - x_2^2 x_3 - x_2^2 + x_2 + 1 = 0 \vee \\ x_2^3 x_3 x_4 - x_2^2 x_3 x_4 - x_2^2 x_3 - x_2^2 x_4 + x_2 x_3 + x_2 x_4 + x_2 - x_4 = 0) \wedge \\ (x_2^3 x_4 - x_2^2 x_4 - x_2^2 + x_2 + 1 = 0 \vee \\ x_2^3 x_3 x_4 - x_2^2 x_3 x_4 - x_2^2 x_3 - x_2^2 x_4 + x_2 x_3 + x_2 x_4 + x_2 - x_3 = 0)\Big).\end{aligned}$$

Related Work

author	application	method
Dolzmann et al.[4]	projection order for CAD	statistical analysis
Huang et al.[5]	projection order for CAD	machine learning
Huang et al.[6]	GB preconditioning for CAD	machine learning

[4] Dolzmann, A., Seidl, A., Sturm, T.: Efficient projection orders for CAD. In: Proceedings of the 2004 international symposium on Symbolic and algebraic computation. pp. 111–118. ACM (2004)

[5] Huang, Z., England, M., Wilson, D., Davenport, J.H., Paulson, L.C., Bridge, J.: Applying machine learning to the problem of choosing a heuristic to select the variable ordering for cylindrical algebraic decomposition. In: Intelligent Computer Mathematics, pp. 92–107. Springer (2014)

[6] Huang, Z., England, M., Davenport, J.H., Paulson, L.C.: Using machine learning to decide when to precondition cylindrical algebraic decomposition with groebner bases. In: Symbolic and Numeric Algorithms for Scientific Computing (SYNASC), 2016 18th International Symposium on. pp. 45–52. IEEE (2016)

Outline

Sorting Subformulas

The following procedure of QE computation stated to be effective[7] in QE for redundant formulas:

1. rearranging an input formula into non-prenex form,
2. sorting over the subformulas,
3. making QE computation for the prior subformula,
4. simplifying the remaining subformulas using the obtained QE result, and

repeating the steps 3 and 4 until computation for all subformulas are finished.

[7] Iwane, H., Matsuzaki, T., Arai, N., Anai, H.: Automated natural language geometry math problem solving by real quantifier elimination. In: Proceedings of the 10th International Workshop on Automated Deduction in Geometry. pp. 75–84 (2014)

input:

$$\exists x_0 \exists x_1 \exists x_2 \exists x_3 \exists x_4 \exists x_5 \Big($$

$$(-x_0 \leq -1) \vee$$
$$(x_1 \leq 0) \vee$$
$$\Big(((x_2^3 x_4 x_5 - x_2^2 x_4 - x_2^2 x_5 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_4 x_5 x_6 - x_2^2 x_4 x_5 - x_2^2 x_4 x_6 - x_2^2 x_5 x_6 + x_2 x_4 + x_2 x_5 + x_2 x_6 - x_6 = 0) \ \wedge$$
$$(x_2^3 x_4 x_6 - x_2^2 x_4 - x_2^2 x_6 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_4 x_5 x_6 - x_2^2 x_4 x_5 - x_2^2 x_4 x_6 - x_2^2 x_5 x_6 + x_2 x_4 + x_2 x_5 + x_2 x_6 - x_5 = 0)) \wedge$$
$$(x_2^3 x_3 - x_2^2 x_3 - x_2^2 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_3 x_4 - x_2^2 x_3 x_4 - x_2^2 x_3 - x_2^2 x_4 + x_2 x_3 + x_2 x_4 + x_2 - x_4 = 0) \wedge$$
$$(x_2^3 x_4 - x_2^2 x_4 - x_2^2 + x_2 + 1 = 0 \vee$$
$$x_2^3 x_3 x_4 - x_2^2 x_3 x_4 - x_2^2 x_3 - x_2^2 x_4 + x_2 x_3 + x_2 x_4 + x_2 - x_3 = 0)\Big)$$

$$\Big).$$

non-prenex form:

$$\varphi_1 \vee \varphi_2 \vee \varphi_3,$$

where

$$\begin{aligned}
&\varphi_1 \equiv \exists x_0(-x_0 \leq -1),\\
&\varphi_2 \equiv \exists x_1(x_1 \leq 0),\\
&\varphi_3 \equiv \exists x_2 \exists x_3 \exists x_4 \Big(\exists x_5 ((x_2^3 x_4 x_5 - x_2^2 x_4 - x_2^2 x_5 + x_2 + 1 = 0 \vee\\
&x_2^3 x_4 x_5 x_6 - x_2^2 x_4 x_5 - x_2^2 x_4 x_6 - x_2^2 x_5 x_6 + x_2 x_4 + x_2 x_5 + x_2 x_6 - x_6 = 0) \;\wedge\\
&\qquad (x_2^3 x_4 x_6 - x_2^2 x_4 - x_2^2 x_6 + x_2 + 1 = 0 \vee\\
&\quad x_2^3 x_4 x_5 x_6 - x_2^2 x_4 x_5 - x_2^2 x_4 x_6 - x_2^2 x_5 x_6 + x_2 x_4 + x_2 x_5 + x_2 x_6 - x_5 = 0)) \wedge\\
&\qquad (x_2^3 x_3 - x_2^2 x_3 - x_2^2 + x_2 + 1 = 0 \vee\\
&\qquad x_2^3 x_3 x_4 - x_2^2 x_3 x_4 - x_2^2 x_3 - x_2^2 x_4 + x_2 x_3 + x_2 x_4 + x_2 - x_4 = 0) \wedge\\
&\qquad (x_2^3 x_4 - x_2^2 x_4 - x_2^2 + x_2 + 1 = 0 \vee\\
&\qquad x_2^3 x_3 x_4 - x_2^2 x_3 x_4 - x_2^2 x_3 - x_2^2 x_4 + x_2 x_3 + x_2 x_4 + x_2 - x_3 = 0)\Big).
\end{aligned}$$

sorting over subformulas φ_1, φ_2, and φ_3:

$$\varphi_1 \to \varphi_2 \to \varphi_3.$$

computing QE for φ_1:

$$\text{true} \vee \varphi_2 \vee \varphi_3.$$

$$(\varphi_1 \equiv \exists x_0(-x_0 \leq -1))$$

simplification:

true.

Heuristic Methods

We experimented with the following heuristic methods:

Human-Constructed. The conditions for determining the order are manually identified.
'ADG'[8], 'nvar', 'npoly', 'sotd', 'msotd', 'mtdeg', 'mdeg', 'mterm', 'nvar_rev', and 'npoly_rev'

Random. The order is determined randomly.

Machine Learning. We use an SVM to determine the order.
'<1', . . . , '<55', '<4_opt', '<8_opt', '<30_opt', and '<9_dim'

[8] Iwane, H., Matsuzaki, T., Arai, N., Anai, H.: Automated natural language geometry math problem solving by real quantifier elimination. In: Proceedings of the 10th International Workshop on Automated Deduction in Geometry. pp. 75–84 (2014)

Use of SVMs for Heuristic Methods

Offline Computation (Training)

Feature Vectors. We identified 58 features to characterize formulas: the number of variables, the number of free variables, the number of polynomials, the number of the symbol '$=$', etc.

Labeling. We label a formula $+1$ if the QE computation was solved in less than N seconds, and -1 otherwise. We experimented with N varying 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30 and 55.

The processes scaling, parameter optimization,and training are also performed to generate SVM models.

Use of SVMs for Heuristic Methods

Online Computation (Prediction)

For a given formula, the *decision value* is calculated:

1. The feature vector of the formula is calculated.
2. The feature vector is scaled using the SVM model.
3. An SVM predicts the decision value for the formula using the SVM model.

Decision values behave as a measure of the confidence in a correct prediction. A formula is determined to be small if the decision value is larger.

Outline

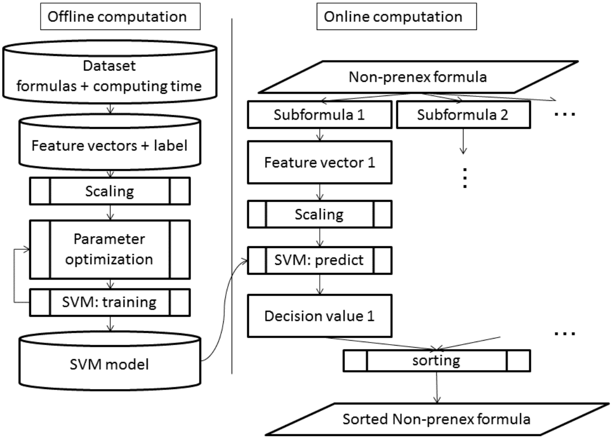
QE Computation (reprint)

The procedure of QE computation:

1. rearranging an input formula into non-prenex form,
2. sorting over the subformulas,
3. making QE computation for the prior subformula,
4. simplifying the remaining subformulas using the obtained QE result, and

repeating the steps 3 and 4 until computation for all subformulas are finished.

Heuristic Methods with SVMs



Outline

QE computations were made using the Maple-package SyNRAC[9].

For the machine learning experiment, we used LIBSVM[10].

We ran all the computational experiments on a computer with an Intel(R) Xeon(R) CPU E7-4870 2.40GHz and 1007 GB of memory.

[9] Iwane, H., Yanami, H., Anai, H.: SyNRAC: A toolbox for solving real algebraic constraints. In: Mathematical Software–ICMS 2014, pp. 518–522. Springer (2014)

[10] Chang, C.C., Lin, C.J.: LIBSVM: A library for support vector machines. ACM Transactions on Intelligent Systems and Technology (TIST) 2(3), 27 (2011)

Datasets

- We have collected 2,306 formulas from the activity of Todai robot project.[11]
- The problems were originally taken from three sources: "Chart-shiki", Japanese university entrance exams, and International Mathematical Olympiads. "Chart-shiki" is a popular problem book series.
- The collected problems were mainly in algebra, linear algebra, and geometry.

[11] Matsuzaki, T., Iwane, H., Kobayashi, M., Zhan, Y., Fukasaku, R., Kudo, J., Anai, H., Arai, N.H.: Race against the teens–benchmarking mechanized math on pre-university problems. In: International Joint Conference on Automated Reasoning. pp. 213–227. Springer (2016)

Experiment

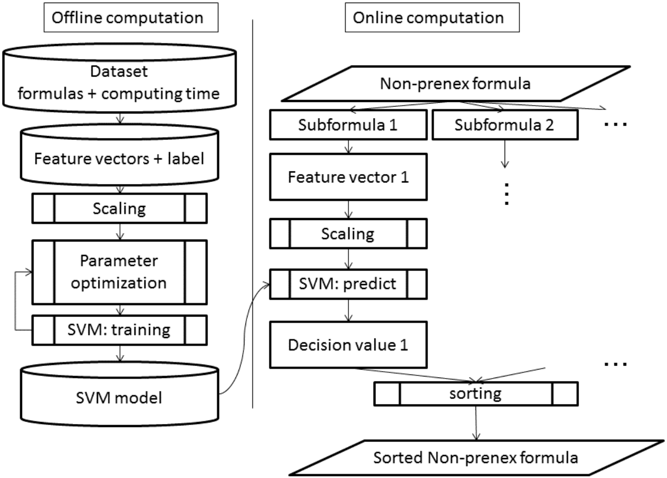
We performed QE for the 2,306 formulas with the 28 prepared heuristic methods.

type	number of heuristics	names
human-constructed	10	‘ADG’, ‘nvar’, ‘npoly’, ‘sotd’, ‘msotd’, ‘mtdeg’, ‘mdeg’, ‘mterm’, ‘nvar_rev’, ‘npoly_rev’
random	1	‘rand‘
machine learning	17	‘<1’, . . . , ‘<55’, ‘<4_opt’, ‘<8_opt’, ‘<30_opt’, ‘<9_dim’

The offline computations were performed to generate 17 SVM models for 2,302 formulas.

Heuristic Methods Configured with SVMs

names	notes
'<1', . . . , '<55'	varied by threshold times for labeling
'<4_opt', '<8_opt', '<30_opt'	modified parameter selection
'<9_dim'	trained with 43 features


Offline computation
Dataset
formulas + computing time
Feature vectors + label
Scaling
Parameter optimization
SVM: training
SVM model
Online computation
Non-prenex formula
Subformula 1
Subformula 2
Feature vector 1
Scaling
SVM: predict
Decision value 1
sorting
Sorted Non-prenex formula

Statistics on Results of Experiment (Human-Constructed and Random)

	solved	error	timeout	aver.(sec)	aver.(log)	median(sec)
ADG	2116	3	187	4.770	−1.038	0.337
nvar	2115	4	187	4.741	−0.993	0.358
npoly	2114	4	188	4.594	−1.134	0.267
sotd	2114	4	188	4.269	−1.217	0.234
msotd	2114	4	188	4.240	−1.212	0.233
mtdeg	2114	4	188	4.172	−1.189	0.262
mdeg	2115	3	188	4.005	−1.230	0.222
mterm	2114	4	188	4.982	−1.150	0.256
nvar_rev	2097	2	207	4.857	−1.129	0.260
npoly_rev	2098	2	206	4.560	−1.119	0.268
rand_aver	2104.93	3.64	197.43	4.764	−1.182	0.241
rand_best	2121	2	183	4.569	−1.389	0.190
rand_worst	2087	2	217	4.531	−0.890	0.356

Statistics on Results of Experiment (Human-Constructed and Random)

	solved	error	timeout	aver.(sec)	aver.(log)	median(sec)
ADG	2116	3	187	4.770	−1.038	0.337
nvar	2115	4	187	4.741	−0.993	0.358
npoly	2114	4	188	4.594	−1.134	0.267
sotd	2114	4	188	4.269	−1.217	0.234
msotd	2114	4	188	4.240	−1.212	0.233
mtdeg	2114	4	188	4.172	−1.189	0.262
mdeg	2115	3	188	4.005	−1.230	0.222
mterm	2114	4	188	4.982	−1.150	0.256
nvar_rev	2097	2	207	4.857	−1.129	0.260
npoly_rev	2098	2	206	4.560	−1.119	0.268
rand_aver	2104.93	3.64	197.43	4.764	−1.182	0.241
rand_best	2121	2	183	4.569	−1.389	0.190
rand_worst	2087	2	217	4.531	−0.890	0.356

Statistics on Results of Experiment (Machine Learning)

	solved	error	timeout	aver.(sec)	aver.(log)	median(sec)
<1	2099	4	203	5.141	-0.934	0.300
<2	2102	3	201	5.320	-0.995	0.332
<3	2115	3	188	4.673	-0.928	0.360
<4	2112	4	190	4.732	-0.959	0.384
<4_opt	2118	2	186	5.020	-1.085	0.279
<5	2117	3	186	5.153	-0.928	0.392
<6	2115	2	189	4.540	-1.155	0.261
<7	2115	2	189	4.508	-1.178	0.244
<8	2113	3	190	4.806	-1.105	0.267
<8_opt	2117	2	187	4.714	-1.142	0.245
<9	2118	2	186	4.750	-1.165	0.239
<9_dim	2116	4	186	4.718	-0.984	0.363
<10	2117	2	187	4.578	-0.956	0.362
<20	2116	3	187	4.451	-1.200	0.235
<30	2114	2	190	4.886	-1.173	0.245
<30_opt	2115	3	188	4.628	-1.199	0.234
<55	2114	3	189	4.808	-0.993	0.351

Statistics on Results of Experiment (Machine Learning)

	solved	error	timeout	aver.(sec)	aver.(log)	median(sec)
<1	2099	4	203	5.141	-0.934	0.300
<2	2102	3	201	5.320	-0.995	0.332
<3	2115	3	188	4.673	-0.928	0.360
<4	2112	4	190	4.732	-0.959	0.384
<4_opt	2118	2	186	5.020	-1.085	0.279
<5	2117	3	186	5.153	-0.928	0.392
<6	2115	2	189	4.540	-1.155	0.261
<7	2115	2	189	4.508	-1.178	0.244
<8	2113	3	190	4.806	-1.105	0.267
<8_opt	2117	2	187	4.714	-1.142	0.245
<9	2118	2	186	4.750	-1.165	0.239
<9_dim	2116	4	186	4.718	-0.984	0.363
<10	2117	2	187	4.578	-0.956	0.362
<20	2116	3	187	4.451	-1.200	0.235
<30	2114	2	190	4.886	-1.173	0.245
<30_opt	2115	3	188	4.628	-1.199	0.234
<55	2114	3	189	4.808	-0.993	0.351

Statistics on Results of Experiment (Machine Learning)

	solved	error	timeout	aver.(sec)	aver.(log)	median(sec)
<1	2099	4	203	5.141	−0.934	0.300
<2	2102	3	201	5.320	−0.995	0.332
<3	2115	3	188	4.673	−0.928	0.360
<4	2112	4	190	4.732	−0.959	0.384
<4_opt	2118	2	186	5.020	−1.085	0.279
<5	2117	3	186	5.153	−0.928	0.392
<6	2115	2	189	4.540	−1.155	0.261
<7	2115	2	189	4.508	−1.178	0.244
<8	2113	3	190	4.806	−1.105	0.267
<8_opt	2117	2	187	4.714	−1.142	0.245
<9	2118	2	186	4.750	−1.165	0.239
<9_dim	2116	4	186	4.718	−0.984	0.363
<10	2117	2	187	4.578	−0.956	0.362
<20	2116	3	187	4.451	−1.200	0.235
<30	2114	2	190	4.886	−1.173	0.245
<30_opt	2115	3	188	4.628	−1.199	0.234
<55	2114	3	189	4.808	−0.993	0.351

Statistics on Results of Experiment (Machine Learning)

	solved	error	timeout	aver.(sec)	aver.(log)	median(sec)
<1	2099	4	203	5.141	-0.934	0.300
<2	2102	3	201	5.320	-0.995	0.332
<3	2115	3	188	4.673	-0.928	0.360
<4	2112	4	190	4.732	-0.959	0.384
<4_opt	2118	2	186	5.020	-1.085	0.279
<5	2117	3	186	5.153	-0.928	0.392
<6	2115	2	189	4.540	-1.155	0.261
<7	2115	2	189	4.508	-1.178	0.244
<8	2113	3	190	4.806	-1.105	0.267
<8_opt	2117	2	187	4.714	-1.142	0.245
<9	2118	2	186	4.750	-1.165	0.239
<9_dim	2116	4	186	4.718	-0.984	0.363
<10	2117	2	187	4.578	-0.956	0.362
<20	2116	3	187	4.451	-1.200	0.235
<30	2114	2	190	4.886	-1.173	0.245
<30_opt	2115	3	188	4.628	-1.199	0.234
<55	2114	3	189	4.808	-0.993	0.351

Parameters of Machine Learning

	<1	<2	<3	<4	<5	<6	<7	<8	<9	<10	<20	<30	<55
$\log_2 C$	12	15.4	9.0	5.4	9.6	11.6	10.8	3.4	10.0	10.8	15.2	7.2	11.8
$\log_2 \gamma$	-3.6	-4.8	-3.0	0.8	-1.8	-2.2	-3.0	1.0	-1.0	-0.8	-2.2	0.8	-1.4
nSV	538	427	428	462	374	354	390	492	324	297	276	350	274

	<4_opt	<8_opt	<30_opt	<9_dim
$\log_2 C$	10.4	9.2	10.6	13.4
$\log_2 \gamma$	-1.2	-0.8	-0.6	0
nSV	359	325	285	292

Statistics on Results of Experiment (Machine Learning)

	solved	error	timeout	aver.(sec)	aver.(log)	median(sec)
<1	2099	4	203	5.141	-0.934	0.300
<2	2102	3	201	5.320	-0.995	0.332
<3	2115	3	188	4.673	-0.928	0.360
<4	2112	4	190	4.732	-0.959	0.384
<4_opt	2118	2	186	5.020	-1.085	0.279
<5	2117	3	186	5.153	-0.928	0.392
<6	2115	2	189	4.540	-1.155	0.261
<7	2115	2	189	4.508	-1.178	0.244
<8	2113	3	190	4.806	-1.105	0.267
<8_opt	2117	2	187	4.714	-1.142	0.245
<9	2118	2	186	4.750	-1.165	0.239
<9_dim	2116	4	186	4.718	-0.984	0.363
<10	2117	2	187	4.578	-0.956	0.362
<20	2116	3	187	4.451	-1.200	0.235
<30	2114	2	190	4.886	-1.173	0.245
<30_opt	2115	3	188	4.628	-1.199	0.234
<55	2114	3	189	4.808	-0.993	0.351

Observations on Experiment

- An inappropriate labeling for training data causes inefficiency of QE computation.
- An inappropriate choice of parameters for SVMs causes inefficiency of QE computation.
- There are ineffective features in the identified 58 features.
- The heuristic methods '$<$9' solved more problems in 600 seconds than 'ADG'.

Outline

Conclusion

- Ordering of subformulas affects the performance of QE computation for non-prenex formulas.
- The human-constructed heuristics performed well compared to random baseline.
- The orderings by machine learning performed still better.
- An inappropriate labeling for training data and choice of parameters for SVMs caused inefficiency of QE computation.
- Machine learning can save a lot of effort to design heuristics by trials and errors.

Thank You for Listening!